Neural Network Training with Approximate Logarithmic Computations[1]

International Conference on Acoustics, Speech and Signal Processing

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[1] Supported in part by the National Science Foundation (CCF-1763747).

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  - All NN operations needs to be defined in LNS
  - In LNS multiplications are cheap but addition are computationally expensive
  - Resort to **Approximate Logarithmic Fixed-Point Computations**

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[2] DOI: 10.1109/TC.1979.1675442

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[3] **DOI: 10.1109/ICC.1995.524253**
[4] **DOI: 10.1109/ARITH.2011.15**
[5] **DOI: 10.1109/FPT.2006.270342**
[6] **DOI: 10.1049/el:19710039**

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- Explored in context of back-propagation but prior to resurgence of neural networks[7]

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- Log-encoding on posits relying on conversions to and from linear domain to perform additions[10]
- LNS circuit implementation for inference on pre-trained recurrent neural network[11]

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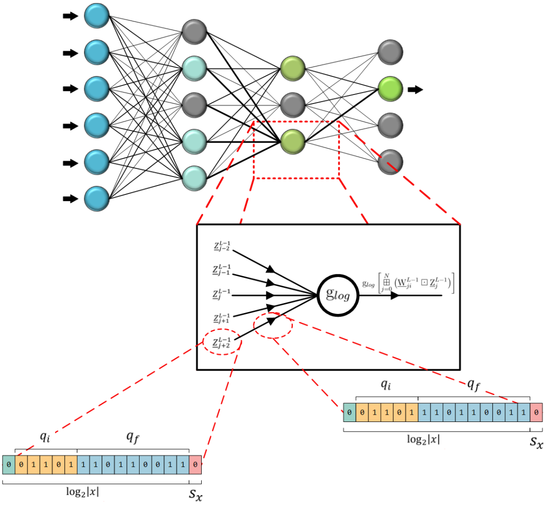
[11] **DOI: 10.1109/MOCAST.2018.8376572**

LNS Neural Network Pipeline

Neural Network Training with Approximate Logarithmic Computations



Equivalence

$$
\begin{aligned}
v \longleftrightarrow \underline{V} &= (V, s_v) \\
V &= \log_2(|v|) \\
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Multiplication

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\begin{aligned}
u = xy \longleftrightarrow \underline{U} &= \underline{X} \boxdot \underline{Y} \\
U &= X + Y \\
s_u &= \overline{(s_x \veebar s_y)}
\end{aligned}
$$

Addition

$$z = x + y \longleftrightarrow \underline{Z} = \underline{X} \boxplus \underline{Y}$$

$$Z = \begin{cases} \max(X, Y) + \Delta_+ \left(|X - Y|\right) & s_x = s_y \\ \max(X, Y) + \Delta_- \left(|X - Y|\right) & s_x \neq s_y \end{cases}$$

$$s_z = \begin{cases} s_x & X > Y \\ s_y & X \leq Y \end{cases}$$

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$$s_z = \begin{cases} s_x & X > Y \\ s_y & X \leq Y \end{cases}$$

$$\Delta_{+}(d) = \log_2\left(1 + 2^{-d}\right) \qquad d \geq 0$$

$$\Delta_{-}(d) = \log_2\left(1 - 2^{-d}\right) \qquad d \geq 0$$

Subtraction

$$t = x - y \longleftrightarrow \underline{T} = \underline{X} \boxminus \underline{Y} = \underline{X} \boxplus (\underline{Y}, \overline{s_y})$$

Mathematical Operations

Neural Network Training with Approximate Logarithmic Computations

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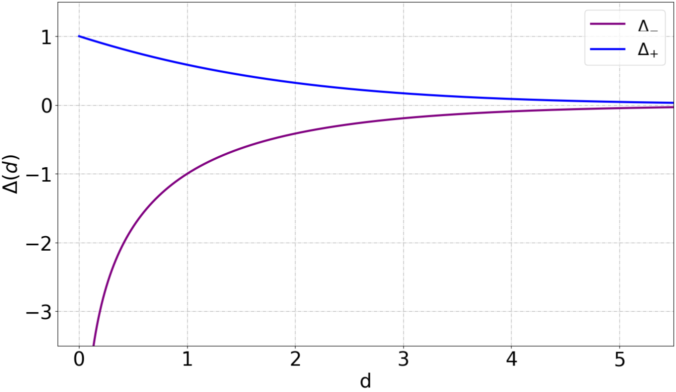
  - Look-Up Table (LUT) based approximations
  - Bit-shift (BS) based approximations

Visualizing Δ from LNS ⊞ Additions

Neural Network Training with Approximate Logarithmic Computations



Look-Up Table Based Approximations

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- Idea - Store the Δ terms as a Look-Up Table (LUT)

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  - Fixed-point width is a hyper-parameter

Look-Up Table Based Approximations

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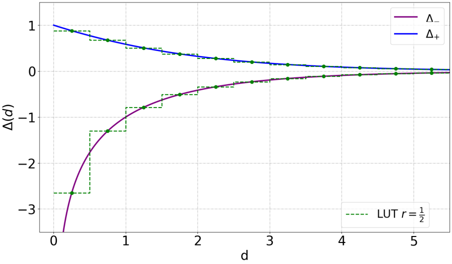

LUT Resolution $r = \frac{1}{2}$

Look-Up Table Based Approximations

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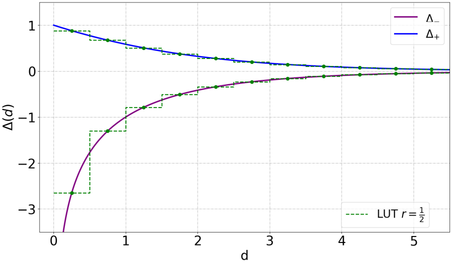

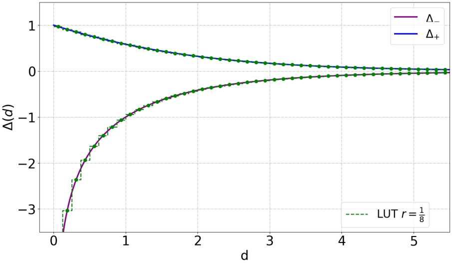

LUT Resolution $r = \frac{1}{2}$

LUT Resolution $r = \frac{1}{8}$

Bit-Shift Based Approximations

Neural Network Training with Approximate Logarithmic Computations

Taylor Series approximation

$$\log_e(1 \pm x) \approx \pm x \qquad 0 \le x \ll 1$$

$$\Delta_{\pm}(d) = \log_2\left(1 \pm 2^{-d}\right) \approx \pm \log_2 e \times 2^{-d}$$

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Further Simplification

$$\log_2 e = 1.442695\cdots \approx 1.4375 = 2^0 + 2^{-1} - 2^{-4} \approx 1.5 = 2^0 + 2^{-1} \approx 1 = 2^0$$

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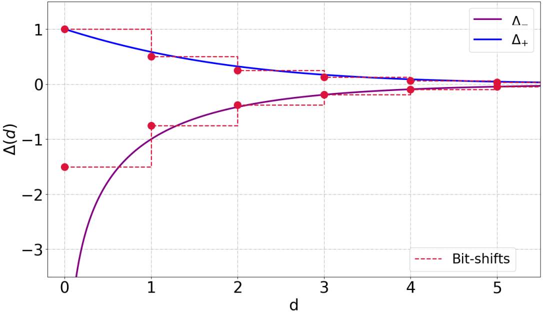

$$\Delta_+(d) \approx \mathbb{BS}\,(1, -d)$$
$$\Delta_-(d) \approx -\mathbb{BS}\,(1.5, -d)$$

Can be thought of as a LUT with $r = 1$

Multiply accumulate
(MAC)

The LNS Neuron

Neural Network Training with Approximate Logarithmic Computations

Multiply accumulate (MAC)

$$z_i = \sum_j w_{ij} x_j + b_i \longleftrightarrow \underline{Z}_i = \boxplus_j \underline{W}_{ij} \boxdot \underline{X}_j \boxplus \underline{B}_i$$

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LNS bit-width Constraint

$$W_{\mathrm{log}} \geq 1 + \max\left(\left\lceil \log_2\left(b_i + 1\right)\right\rceil, \left\lceil \log_2 b_f \right\rceil\right) + W_{\mathrm{lin}}$$

Experiments suggest that $W_{\mathrm{log}} \approx W_{\mathrm{lin}}$ suffices in practice. In this work, set $W_{\mathrm{log}} = W_{\mathrm{lin}}$

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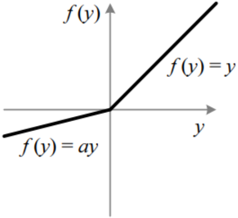
- Weights generally initialized from symmetric distributions. Hence the sign parameter can be initialized randomly and independently of the magnitude from a $Bernoulli(\frac{1}{2})$ distribution.
- The magnitude distribution for weights reduce to,

$$f_W(y) = 2^{y+1} \times \log_e 2 \times f_w(2^y)$$

The Log-Leaky ReLU Activation

Neural Network Training with Approximate Logarithmic Computations



Parametric ReLU

The Log-Leaky ReLU Activation

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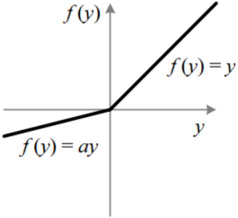

Parametric ReLU

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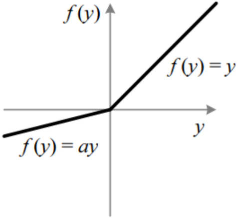

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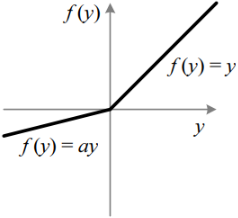

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- This brings us to *Log-Leaky ReLU*

$$g_{\text{llReLU}}\left((X, s_x)\,\middle|\,\beta\right) = \begin{cases} (X, s_x) & s_x = 1 \\ (X + \beta, s_x) & s_x = 0 \end{cases}$$

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LNS Soft-Max

Gradient calculation

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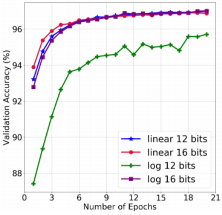
LNS gradients

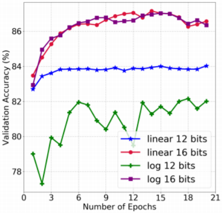
$$\left(\log_2 |\delta_{ij}|, s_{\delta_{ij}}\right) = \underline{P}_{ij} \boxminus \left(\log_2 |y_{ij}|, s_{y_{ij}}\right)$$

Numerical Results

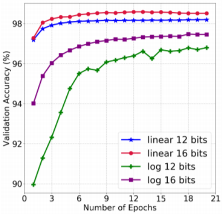
Neural Network Training with Approximate Logarithmic Computations



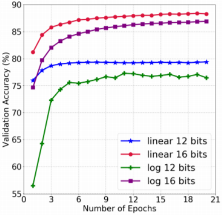
(a) MNIST



(b) Fashion-MNIST



(c) EMNIST-digits



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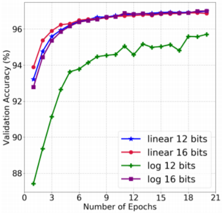

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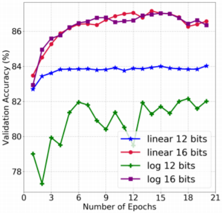

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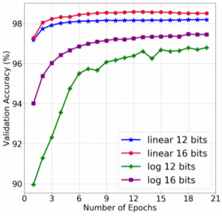

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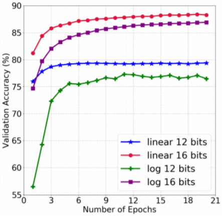

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Datasets	Float	**Linear-domain** fixed-point		**Log-domain** fixed-point look-up tables		**Log-domain** fixed-point bit-shifts	
		12b	16b	12b	16b	12b	16b
MNIST	97.4	97.3	96.9	96.0	97.2	95.5	96.5
FMNIST	87.1	82.8	88.0	80.5	87.1	79.3	85.7
EMNISTD	98.6	98.3	98.7	96.9	97.5	96.2	97.4
EMNISTL	88.1	79.7	88.7	76.4	86.7	73.7	82.5

Number of epochs trained $= 20$
Size of tables $= 20(r = \frac{1}{2})$; soft-max uses 640 element tables$(r = \frac{1}{64})$
Table shows test-set accuracy

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  - Weight-Activation Relation mapping for LNS-neurons using Supervised Learning (LDA, QDA)

Acknowledgements

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